

The Correlated Lloyd model: exact solution

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February 25, 2014

Abstract

Exactly solvable model of disordered system representing the generalized Lloyd model with correlated random potential is described. It is shown, that for the model under consideration, the averaged Green's function does not depend on random potential correlation radius and, similarly to the classical Lloyd model, has the form of Green's function of a crystal system, with energy argument supplied by an imaginary part which depends on degree of disorder.

Introduction

The number of annually published papers devoted to the models of disordered systems, has noticeably increased during the last decade, The "center of gravity" of up to date research is shifting towards *correlated* disordered systems, exploration of which becomes more and more popular [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In the historically first models of random systems, as a rule, the δ -correlated random potential was employed and relatively small attention was paid to the influence of correlations. The recent research have shown, that correlations can lead to a considerable (sometimes, qualitative) modification of energy structure and localisation properties of disordered systems.

The exactly solvable models play an important role in an arbitrary field of theoretical physics. Exactly solvable model allows one to accumulate the qualitative information related to the appropriate class of models with a high degree of reliability, to test used approximations, to specify a direction of further research, etc. The most known and important exactly solvable models in physics of disordered systems are Dyson model [11] and Lloyd model [12]. Both these models relate to the uncorrelated disorder. In this paper we propose a generalized one-dimensional Lloyd model, with *site energies being not an independent random variables* and describe the exact calculation of the averaged Green's function, and show, that it does not depend on the model's parameter which plays the role of correlation radius.

The paper is organised as follows. In the first section we consider the classical Lloyd model, formulate definitions, necessary for further analysis and obtain some auxiliary results. In the second section we introduce the correlated disorder, for which the exact evaluation of the averaged Green's function is carried out in the third section.

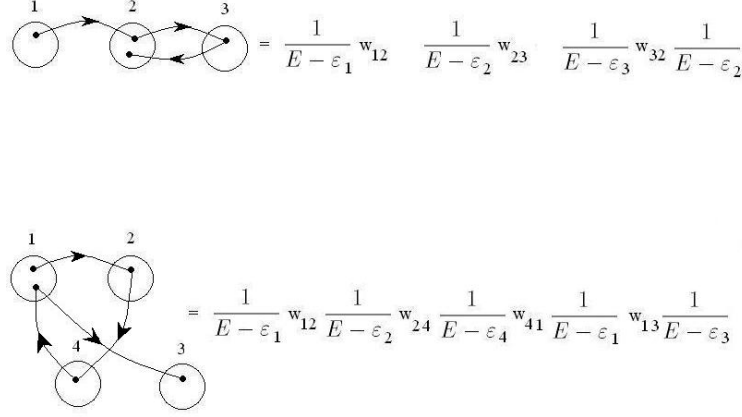


Figure 1: Examples of the diagrams

1 Lloyd model

Matrix \mathbf{H} of the Hamiltonian of classical one-dimensional Lloyd model has the following elements:

$$H_{rr'} = \delta_{rr'} \varepsilon_r + w(r - r') \quad r, r' = 1, 2, 3, \dots, N \quad (1)$$

where function $w(r)$ (with $w(0) = 0$) is specified, and diagonal elements (the site energies) ε_r are independent, similarly distributed random variables with Cauchy distribution function

$$P(\varepsilon) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + \varepsilon^2} \quad (2)$$

Parameter Δ characterizes a degree of disorder – at $\Delta \rightarrow 0$ the Hamiltonian (1) corresponds to ordered (crystalline) system and can be diagonalised in the representation of plane waves. A set of numbered site energies is frequently referred to as a random potential.

In his famous work [12] Lloyd has managed to calculate exactly the averaged Green's function $\langle \mathbf{G}(\Omega) \rangle = \langle [\Omega - \mathbf{H}]^{-1} \rangle$, ($\Omega \equiv E + i0$) for a model system (1), (2), which (Green function) allows one to calculate density of states and spectrum of linear susceptibility. In this section we reproduce Lloyd's result by means of a dyagram technique, similar to that offered in [13] (see also [14]). Let's introduce a matrix \mathbf{W} with elements $W_{rr'} \equiv w(r - r')$. Then the Green's function matrix \mathbf{G} can be written as the following series [15]

$$G_{rr'}(\Omega) = \frac{\delta_{rr'}}{\Omega - \varepsilon_r} + \frac{1}{\Omega - \varepsilon_r} W_{rr'} \frac{1}{\Omega - \varepsilon'_r} + \sum_{r''} \frac{1}{\Omega - \varepsilon_r} W_{rr''} \frac{1}{\Omega - \varepsilon''_r} W_{r''r'} \frac{1}{\Omega - \varepsilon_r} + \dots \quad (3)$$

Let's denote matrix element $W_{r,r'}$ by an arrow, directed from site r to site r' and denote factor $[\Omega - \varepsilon_r]^{-1}$, related to the site r , by a bold dot, placed inside the appropriate site. The examples are presented at fig.1. Then one can write the following expression for the matrix element $G_{rr'}(\Omega)$ of the Green's function

$$G_{rr'}(\Omega) = \text{the sum of all diagrams pairing sites } r \text{ and } r' \quad (4)$$

To calculate the sought averaged Green's function one should integrate (4) with a joint distribution function of site energies $\varepsilon_1, \dots, \varepsilon_N$. In the case of uncorrelated disorder this function can be represented as a product:

$$\rho_{nc}(z_1, z_2, \dots, z_N) = \prod_{i=1}^N P(z_i), \quad (5)$$

For Lloyd model under consideration the function $P(z)$ has the form (2). Averaging of an arbitrary diagram D in expansion (4), is reduced to averaging of factor f_D defined as:

$$f_D \equiv \left(\frac{1}{\Omega - \varepsilon_{n_1}} \right)^{g_1} \left(\frac{1}{\Omega - \varepsilon_{n_2}} \right)^{g_2} \dots \left(\frac{1}{\Omega - \varepsilon_{n_q}} \right)^{g_q} \quad (6)$$

where n_1, n_2, \dots, n_q – are the numbers of sites, the diagram D passed through, and g_i is the number of diagram's passages through the site $n_i, i = 1, 2, \dots, q$ (number of bold dots inside the site n_i). For example, for upper diagram at fig.1 we have: $n_1 = 1, n_2 = 2, n_3 = 3$ and $g_1 = 1, g_2 = 2, g_3 = 1$. The key point for Lloyd's solution is the following rule for calculating of relevant integrals

$$\frac{1}{\pi} \int \frac{\Delta dz}{\Delta^2 + z^2} \left(\frac{1}{E + i0 - z} \right)^n = \left(\frac{1}{E + i\Delta} \right)^n \quad (7)$$

Taking advantage of this relationship and implying the site energies $\varepsilon_{n_i}, i = 1, 2, \dots, q$ to be mutually independent, we obtain the following expression for the averaged factor f_D :

$$\begin{aligned} \langle f_D \rangle &= \int \prod_{i=1}^N dz_i P(z_i) \left(\frac{1}{\Omega - z_{n_1}} \right)^{g_1} \left(\frac{1}{\Omega - z_{n_2}} \right)^{g_2} \dots \left(\frac{1}{\Omega - z_{n_q}} \right)^{g_q} = \\ &= \left(\frac{1}{\Omega + i\Delta} \right)^{g_1} \left(\frac{1}{\Omega + i\Delta} \right)^{g_2} \dots \left(\frac{1}{\Omega + i\Delta} \right)^{g_q} \end{aligned} \quad (8)$$

which coincides with that for a diagram D of the Green's function of the *ordered* system with all site energies being zero, and with energy argument being replaced as: $\Omega \rightarrow \Omega + i\Delta$. The above calculation is valid for any diagram in expansion (4) and we come to the Lloyd's result: *the averaged Green function of disordered system described by the Hamiltonian (1) with the Cauchy uncorrelated disorder (2) is equal to a Green's function \mathbf{G}^{od} of the ordered system with zero site energies, and in which the replacement $\Omega \rightarrow \Omega + i\Delta$ of energy argument is made:*

$$\langle \mathbf{G}(\Omega) \rangle = \mathbf{G}^{od}(\Omega + i\Delta) \quad (9)$$

The explicit expression for a Green's function matrix of ordered (and cyclic) system \mathbf{G}^{od} can be obtained by using the fact that eigenvectors of the Hamiltonian (1) at $\varepsilon_r = 0$ ($r = 1, \dots, N$) are plane waves [15]. For the case of one-dimensional system this matrix is:

$$G_{rr'}^{od}(\Omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i(r-r')q}}{\Omega - J_q} dq, \quad J_q = \sum_r w(r) e^{-iqr} \quad (10)$$

In the next section we describe the simple correlated discrete random process $\varepsilon_1, \dots, \varepsilon_N$, for which the total joint distribution function $\rho(z_1, z_2, \dots, z_N)$ (which can not be reduced to the product like (5)) can be calculated in the close form. In the final, third, section we show, that for the random Hamiltonian (1), with site energies represented by such random process, the result (9) holds.

2 Correlated disorder

We obtain the correlated sequence of site energies $\varepsilon_r, r = 1, \dots, N$ by the following procedure of *smoothing*¹ [16, 17]. Let's introduce a set of independent random values $\xi_i, i = -\infty, \dots, -1, 0, 1, \dots, +\infty$, each of which has the specified distribution function $P(\xi)$ (at this stage of calculation it can differ from (2)), independent on i . Now obtain the site energies ε_n as a realisation of the following discrete random process:

$$\varepsilon_n = (1 - e^{-\alpha}) \sum_{m \leq n} e^{\alpha(m-n)} \xi_m, \quad \alpha > 0 \quad n = 1, \dots, N \quad (11)$$

The values ε_n will be correlated with a correlation radius $R = 1/\alpha$. The relevant correlation function $\langle \varepsilon_n \varepsilon_{n'} \rangle$ has the form

$$\langle \varepsilon_n \varepsilon_{n'} \rangle = \langle \xi^2 \rangle \left(\frac{1 - e^{-\alpha}}{1 + e^{-\alpha}} \right) e^{-\alpha|n-n'|} = \langle \xi^2 \rangle \left(\frac{1 - \beta}{1 + \beta} \right) e^{-\alpha|n-n'|}, \quad \beta \equiv e^{-\alpha}, \quad \beta \in [0, 1] \quad (12)$$

From definition (11) it is easy to obtain the following (important for the further) relationship:

$$\varepsilon_{n+1} = \beta \varepsilon_n + (1 - \beta) \xi_{n+1} \quad (13)$$

The correlation function (12) is meaningful only for the case of finite second moment of the function $P(\xi)$ and is not exist if $P(\xi)$ has the form (2). Nevertheless, even in this case it is not correct to consider a sequence (11) as uncorrelated, because, as we shall see below, its joint distribution function can not be presented in the form (5). At last, we note, that the correlated sequence (11) is causal – i.e. ε_n depends only on those ξ_m , for which $m \leq n$.

2.1 Site energy distribution function for the case of correlated process (11)

The distribution function of an arbitrary site energy ε_n (we shall denote it $\sigma(\varepsilon)$) does not depend on its number n and we calculate it for $n = 0$ [16, 17]. As a starting point we use the following general expressions for the sought function $\sigma(\varepsilon)$ and the relevant characteristic function $\tilde{\sigma}(t)$:

$$\sigma(\varepsilon) = \left\langle \delta \left(\varepsilon - [1 - \beta] \sum_{m=0}^{\infty} \beta^m \xi_m \right) \right\rangle \equiv \int e^{i\varepsilon t} \tilde{\sigma}(t) \quad (14)$$

$$\tilde{\sigma}(t) = \frac{1}{2\pi} \left\langle \exp -it \left([1 - \beta] \sum_{m=0}^{\infty} \beta^m \xi_m \right) \right\rangle = \frac{1}{2\pi} \prod_{m=0}^{\infty} \int d\xi P(\xi) e^{-it[1-\beta]\beta^m \xi} \quad (15)$$

Here angular brackets correspond to an averaging over independent auxiliary variables ξ_m . Denoting the Fourier transformation of the function $P(\xi)$ as $\tilde{P}(t)$: $\tilde{P}(t) = \int P(\xi) e^{-it\xi} d\xi$, we obtain the following formula for $\tilde{\sigma}(t)$:

$$\tilde{\sigma}(t) = \frac{1}{2\pi} \prod_{m=0}^{\infty} \tilde{P}(t[1 - \beta]\beta^m) \quad (16)$$

¹the similar mechanism of correlation was employed in [9] for calculation of a degree of localization of states in the correlated system.

If $P(\xi)$ is the Cauchy function (2), then

$$\tilde{P}(t) = e^{-|t|\Delta} \Rightarrow \tilde{\sigma}(t) = \frac{1}{2\pi} \exp\left(-|t|[1-\beta]\Delta \sum_{m=0}^{\infty} \beta^m\right) = \frac{1}{2\pi} e^{-|t|\Delta} \quad (17)$$

Therefore the distribution function of site energies in our case also has the form of Cauchy (2) function

$$\sigma(\varepsilon) = P(\varepsilon) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + \varepsilon^2} \quad (18)$$

2.2 Total distribution function of a random process (11)

The discrete correlated random process ε_n (11) is completely determined by joint distribution function of all site energies $\rho(z_1, z_2, \dots, z_N)$, which can be calculated analytically. For this purpose we shall introduce the joint probability distribution functions of the first M ($0 < M < N$) site energies $\rho_M(z_1, z_2, \dots, z_M)$. So, the value $\rho_M(z_1, z_2, \dots, z_M) dz_1 \dots dz_M$ gives a probability that $\varepsilon_i \in [z_i, z_i + dz_i], i = 1, \dots, M$. Using equation (13) one can obtain the recurrent relationship expressing ρ_{M+1} through ρ_M :

$$\begin{aligned} \rho_{M+1}(z_1, z_2, \dots, z_{M+1}) &= \left\langle \delta(z_1 - \varepsilon_1) \dots \delta(z_M - \varepsilon_M) \delta(z_{M+1} - \varepsilon_{M+1}) \right\rangle = \\ &= \left\langle \delta(z_1 - \varepsilon_1) \dots \delta(z_M - \varepsilon_M) \delta(z_{M+1} - \beta \varepsilon_M - [1 - \beta] \xi_{M+1}) \right\rangle = \\ &= \int d\xi dy_1 \dots dy_M \rho_M(y_1, \dots, y_M) P(\xi) \delta(z_1 - y_1) \dots \delta(z_M - y_M) \delta(z_{M+1} - \beta y_M - [1 - \beta] \xi) = \\ &= \frac{1}{1 - \beta} \rho_M(z_1, \dots, z_M) P\left(\frac{z_{M+1} - \beta z_M}{1 - \beta}\right) \end{aligned} \quad (19)$$

Sequentially applying this relationship and taking into account, that $\rho_1(z) = \sigma(z)$, we obtain the following expression for the function $\rho_M(z_1, \dots, z_M)$

$$\rho_M(z_1, z_2, \dots, z_M) = \frac{1}{[1 - \beta]^{M-1}} \overbrace{P\left(\frac{z_M - \beta z_{M-1}}{1 - \beta}\right) P\left(\frac{z_{M-1} - \beta z_{M-2}}{1 - \beta}\right) \dots P\left(\frac{z_2 - \beta z_1}{1 - \beta}\right)}^{M-1} \sigma(z_1) \quad (20)$$

In our case $\sigma(z) = P(z)$, where $P(z)$ is defined by formula (2). Supposing $M = N$, we obtain the following final expression for the total joint distribution function of random process (11)

$$\rho(z_1, z_2, \dots, z_N) = \overbrace{P\left(\frac{z_N - \beta z_{N-1}}{1 - \beta}\right) P\left(\frac{z_{N-1} - \beta z_{N-2}}{1 - \beta}\right) \dots P\left(\frac{z_2 - \beta z_1}{1 - \beta}\right)}^{N-1} \frac{P(z_1)}{[1 - \beta]^{N-1}} \quad (21)$$

$$P(z) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + z^2}$$

3 Correlated Lloyd model

Our task now is to calculate the averaged Green's function of disordered model described by the Hamiltonian (1), with site energies representing the realisation of correlated random process (11). Let's consider, as well as in the first section, an arbitrary diagram D , passing through the sites n_1, n_2, \dots, n_q , whose numbers without loss of generality we can consider to be arranged in ascending order: $1 \leq n_1 < n_2 < \dots < n_q \leq N$. The average value $\langle f_D \rangle$ of factor (6) of the diagram under consideration is now defined by a formula, differed from (8):

$$\langle f_D \rangle = \int dz_1 dz_2 \dots dz_N \left(\frac{1}{E + i\delta - z_{n_1}} \right)^{g_1} \left(\frac{1}{E + i\delta - z_{n_2}} \right)^{g_2} \dots \left(\frac{1}{E + i\delta - z_{n_q}} \right)^{g_q} \rho(z_1, z_2, \dots, z_N) \quad (22)$$

where function $\rho(z_1, z_2, \dots, z_N)$ for our correlated Lloyd model has the form (21). As $\delta > 0$, the factors in big brackets in the relationship (22), considered as a functions of complex z_{n_1}, \dots, z_{n_q} , have no singularities in a lower half-plane of complex z_{n_1}, \dots, z_{n_q} . This allows us to use formula (7) for evaluation of integrals figuring in the formula (22) as follows.

First of all let's integrate the relationship (22) over all z_i with $n_q < i \leq N$. After that the function (21), entering this relationship, will lose its first $N - n_q$ factors of a type $P\left(\frac{z_N - \beta z_{N-1}}{1 - \beta}\right) P\left(\frac{z_{N-1} - \beta z_{N-2}}{1 - \beta}\right) \dots$ and denominator $[1 - \beta]^{N-1}$ will be replaced by $[1 - \beta]^{n_q-1}$.

Then perform the integration over z_{n_q} , which affects only the Lorentzian $P\left(\frac{z_{n_q} - \beta z_{n_q-1}}{1 - \beta}\right)$, entering (21), and can be carried out with the help of formula (7). This formula shows that mentioned integration corresponds to multiplication by the factor $1 - \beta$ and to replacement $z_{n_q} \rightarrow \beta z_{n_q-1} - i\Delta(1 - \beta)$ in the factor $\left(\frac{1}{E + i\delta - z_{n_q}}\right)^{g_q}$, entering the integrand in (22). Thus the pole of this function (with respect to z_{n_q-1}) will still be placed in the upper half plane. This allows to perform the next integration over z_{n_q-1} in the same manner. The only function entering joint probability density (21) depending on this argument is $P\left(\frac{z_{n_q-1} - \beta z_{n_q-2}}{1 - \beta}\right)$. As well as in the previous case, the integration over z_{n_q-1} corresponds to multiplication by the factor $1 - \beta$ and replacement $z_{n_q-1} \rightarrow \beta z_{n_q-2} - i\Delta(1 - \beta)$ in factor $\left(\frac{1}{E + i\delta - \beta z_{n_q-1} + i\Delta(1 - \beta)}\right)^{g_q}$, which arose as a result of the previous integration, etc. Hence, every new integration over z_i with lower and lower number i corresponds to multiplication by $1 - \beta$ (i.e. erasing of such a factor in the denominator of expression (21)) and to replacement $z_i \rightarrow \beta z_{i-1} - i\Delta(1 - \beta)$ in the last term.

When the number i of a variable of integration will decrease down to $i = n_{q-1}$, the further integrations can be performed similarly (i.e. by making replacements of arguments), but now the above replacements should be performed in the factor $\left(\frac{1}{E + i\delta - z_{n_{q-1}}}\right)^{g_{q-1}}$ as well.

Thus, we come to the conclusion, that integration over all variables in (22) corresponds to sequential replacements of symbols $z_{n_q}, \dots, z_{n_2}, z_{n_1}$ figuring in (22) in accordance with the above rules. For example, the appropriate replacements for z_{n_q} have the form: ²

$$z_{n_q} = \beta z_{n_q-1} - i\Delta(1 - \beta), \text{ where} \quad (23)$$

²As n_q is the major of sites numbers n_i of the diagram under consideration, the replacements of remaining symbols are included in the sequence, presented below.

$$\begin{aligned}
z_{n_q-1} &= \beta z_{n_q-2} - \imath \Delta (1 - \beta), \text{ where} \\
z_{n_q-2} &= \beta z_{n_q-3} - \imath \Delta (1 - \beta), \text{ where...} \\
&\dots \\
z_3 &= \beta z_2 - \imath \Delta (1 - \beta), \text{ where...} \\
z_2 &= \beta z_1 - \imath \Delta (1 - \beta).
\end{aligned}$$

The last integration over z_1 corresponds to replacement $z_1 \rightarrow -\imath \Delta$, as this integration is performed with function $P(z_1)$ (see (21)). It is easy to see, that if $z_1 = -\imath \Delta$, the chain of replacements (23) simplifies and corresponds to the replacement $z_{n_i} = -\imath \Delta, i = 1 \dots, n_q$. Thus, the average (22) corresponds to the replacement of all symbols $z_{n_i}, i = 1 \dots, q$ with $-\imath \Delta$, as well as in the case of uncorrelated Lloyd model (8), and we come to the conclusion, that *the averaged Green's function of the correlated Lloyd model with site energies in the form (11) does not depend on the correlation radius $R = -1/\ln \beta$ and appears to be the same, as at lack of correlations*, i.e. is defined by formula (9).

Lack of dependence of the averaged Green's function on correlation radius $R = -1/\ln \beta$ reveals an original scale invariance of the considered correlated Lloyd model, because the spatial dependence (dependence on the site number) of random potential ε_n (11) appears to be essentially not the same for various R (fig. 2 (d, c)).

Let's illustrate the obtained result by examples, when Green's function $\mathbf{G}^{od}(\Omega)$ (10) can be calculated analytically. First (well known) example of this kind is the case of tight-binding Hamiltonian with appropriate matrix $H_{rr'}^{tb} = \delta_{r,r'+1} + \delta_{r,r'-1}$ and diagonal elements of Green's function defined as $G_{nn}^{od}(\Omega) = [\Omega^2 - 4]^{-1/2}$ [15]. According to the results obtained above, the average density of states $\rho_{\Delta}^{tb}(E) = -\pi^{-1} \text{Im Sp } \langle \mathbf{G} \rangle$ of the random matrix (1) with site energies ε_r in the form (11) and with $w(r) = \delta_{r,1} + \delta_{r,-1}$, does not depend on correlation radius $R = -1/\ln \beta$ and can be calculated as:

$$\rho_{\Delta}^{tb}(E) = -\frac{N}{\pi} \text{Im} \frac{1}{\sqrt{(E + \imath \Delta)^2 - 4}} \quad (24)$$

The second (less known) example is the case, when the matrix of ordered Hamiltonian has the form: $H_{rr'}^{ex} = v_0 \exp -|r - r'|/R_0$. In this case the Green's function matrix is defined by the relationship [18]:

$$\Gamma_{rr'}(\Omega) = A \exp(-|r - r'| \eta) + \frac{\delta_{rr'}}{\Omega} \quad (25)$$

where

$$A \equiv \frac{V}{(\Omega - V) \Omega \sqrt{1 - T^2}}, \quad V \equiv v_0 \text{th} \left(\frac{1}{R_0} \right), \quad \frac{1}{T} \equiv \frac{V - \Omega}{\Omega} \text{ch} \left(\frac{1}{R_0} \right), \quad \text{ch} \eta \equiv \left| \frac{1}{T} \right|$$

and the density of states of random matrix (1) with $w(r) = v_0 \exp -|r - r'|/R_0$ and ε_r (11) is defined by the expression: ³

$$\rho_{\Delta}^{ex}(E) = -\frac{N}{\pi} \text{Im } \Gamma_{00}(E + \imath \Delta + v_0) \quad (26)$$

³The occurrence of shift v_0 in this formula is the consequence of the fact that Green's function (25) is obtained for matrix \mathbf{H}^{ex} whose diagonal elements are nonzero and equal to v_0 .

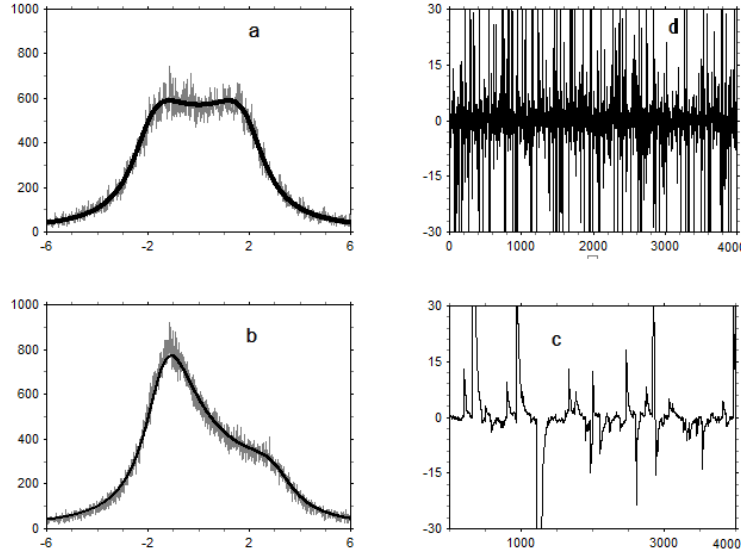


Figure 2: The correlated Lloyd model at various radiuses of correlation R and various types of function $w(r)$, describing the nondiagonal elements of the Hamiltonian (1). Panels (a) and (b) – density of states of the Hamiltonian (1) for $w(r) = \delta_{1,r} + \delta_{-1,r}$ (a), and for $w(r) = v_0 \exp -|r/R_1|$ ($R_1 = 1, v_0 = \exp[1/R_1]$) (b). The noisy dependences on boards (a) and (b) are obtained by numerical diagonalization of the Hamiltonian (1), smooth curves – calculation by formulas (24) and (26) respectively. The densities of states shown on panels (a) and (b) are obtained for correlation radiuses $R = 0.1$ and $R = 30$ respectively. Boards (d) and (c) – realisations of random potentials for correlation radiuses $R = 0.1$ and $R = 30$ respectively. On an abscissa axis the site number n , on an axis of ordinates – ε_n (11) is postponed. In all cases $\Delta = 1$. The size of random matrixes for numerical calculations $N = 4000$.

For these two cases the numerical diagonalization of random matrixes (1) for various correlation radiuses R has shown, that the density of states is described by formulas (24) and (26) (to within noise) and really does not depend on correlation radius R , despite the fact, that random potential (11) reveals strong dependence on R (fig. 2 (d,c)).

Conclusion

The exact calculation of averaged Green's function for the correlated Lloyd model is presented. It is shown, that for a considered type of correlated random potential the averaged Green's function does not depend on parameter of a random potential playing a role of correlation radius. The obtained result is verified by numerical calculations.

Finally we want to emphasize, that the above result is valid for an arbitrary dimensionality of the lattice, if indexing of sites and their random energies satisfying the relationship (11).

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